PHYS 101 Midterm Exam 1 Solution 2021-22-2

1. Two point particles A and B are moving on the xy-plane. At time $t = 0$, particle A is at the point $(x = 20\text{ m}, y = 0)$, and is moving with constant speed 4 m/s in the +x direction, while particle B is at the origin moving with velocity $\vec{\mathbf{v}}_{0B} = 15\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{j}}$ m/s. Particle A has no acceleration, while the acceleration of particle B is $\vec{\mathbf{a}}_B = -10\,\hat{\mathbf{j}}\ \text{m}/s^2$.

(a) (4 Pts.) Write the position vector $\vec{\mathbf{r}}_A$ of particle A and $\vec{\mathbf{r}}_B$ of particle B as a function of time.
(b) (6 Pts.) What is the average velocity of particle B during the time interval from $t = 1s$ to $t = 3s$?
(c) (6 Pts.) What is the distance between the two particles when $t = 1s$?
(d) (6 Pts.) At what point will particle A be when the y coordinate of particle B is zero for $t > 0$?
(e) (8 Pts.) What is the relative velocity of particle B with respect to particle A as a function of time?

Solution:

Given $\vec{\mathbf{r}}_{0A} = 20\,\hat{\mathbf{i}}\ (\text{m})$, $\vec{\mathbf{v}}_{0A} = 4\,\hat{\mathbf{i}}\ (\text{m/s})$, $\vec{\mathbf{a}}_A = 0$, $\vec{\mathbf{r}}_{0B} = 0$, $\vec{\mathbf{v}}_{0B} = 15\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{j}}\ (\text{m/s})$, and $\vec{\mathbf{a}}_B = -10\,\hat{\mathbf{j}}\ (\text{m/s}^2)$.

(a) For motion with constant acceleration $\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_0 + \vec{\mathbf{v}}_0 t + \frac{1}{2}\vec{\mathbf{a}}t^2$. Therefore,

$$\vec{\mathbf{r}}_A(t) = (20 + 4t)\hat{\mathbf{i}}\ (\text{m}), \qquad \vec{\mathbf{r}}_B(t) = (15t)\hat{\mathbf{i}} + (20t - 5t^2)\,\hat{\mathbf{j}}\ (\text{m})\,.$$

(b) $\vec{\mathbf{r}}_B(1) = (15)\hat{\mathbf{i}} + (15)\,\hat{\mathbf{j}}\ (\text{m}), \vec{\mathbf{r}}_B(3) = (45)\hat{\mathbf{i}} + (15)\,\hat{\mathbf{j}}\ (\text{m}) \quad \rightarrow \quad \Delta\vec{\mathbf{r}}_B = \vec{\mathbf{r}}_B(3) - \vec{\mathbf{r}}_B(1) = (30)\hat{\mathbf{i}}\ (\text{m})$

$$\vec{\mathbf{v}}_{\text{B-Av}} = \frac{\Delta\vec{\mathbf{r}}_B}{\Delta t} = \frac{\vec{\mathbf{r}}_B(3) - \vec{\mathbf{r}}_B(1)}{3 - 1} = (15)\hat{\mathbf{i}}\ (\text{m/s}).$$

(c) Distance between the two particles is $D_{AB}(t) = |\vec{\mathbf{r}}_B(t) - \vec{\mathbf{r}}_A(t)|$. Hence, for $t = 1$ s, we have

$$D_{AB}(1) = |\vec{\mathbf{r}}_B(1) - \vec{\mathbf{r}}_A(1)| = |(-9)\hat{\mathbf{i}} + (15)\,\hat{\mathbf{j}}\ (\text{m})| = \sqrt{(-9)^2 + (15)^2}\ (\text{m}) = \sqrt{306}\ (\text{m})\,.$$

(d) The *y*-coordinate of particle B is $y_B = 20t - 5t^2$. Therefore $y_B = 0 \quad \rightarrow \quad t_1 = 0, \text{or}, t_2 = 4\text{ s}$. So

$$\vec{\mathbf{r}}_A(4) = (36)\hat{\mathbf{i}}\ (\text{m}).$$

Particle A will be at the point $(36, 0)$ at time $t = 4$ s.

(e) Velocities of the two particles are

$$\vec{\mathbf{v}}_B(t) = \frac{d\vec{\mathbf{r}}_B}{dt} = (15)\hat{\mathbf{i}} + (20 - 10t)\,\hat{\mathbf{j}}\ (\text{m/s})\,, \qquad \vec{\mathbf{v}}_A(t) = \frac{d\vec{\mathbf{r}}_A}{dt} = (4)\hat{\mathbf{i}}\ (\text{m/s})\,.$$

Hence, relative veocity of particle B with respect to particle A is

$$\vec{\mathbf{v}}_{B/A}(t) = \vec{\mathbf{v}}_B(t) - \vec{\mathbf{v}}_A(t) = (11)\hat{\mathbf{i}} + (20 - 10t)\,\hat{\mathbf{j}}\ (\text{m/s})\,.$$

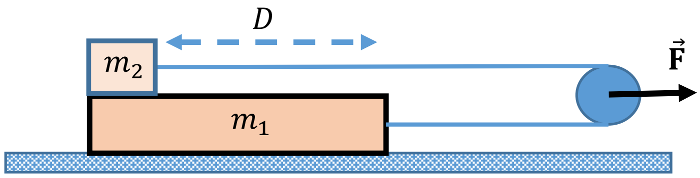
2. A mass m_2 is on top of a platform with mass m_1 (assume $m_2 < m_1$). There is no friction between the mass and the platform, or between the platform and the horizontal floor. The two masses are connected by an ideal massless string going over a massless pulley. Initially m_2 is a distance D away from the right edge of the platform and all masses are at rest. Starting at time $t = 0$, a force F is applied to the pulley horizontally, as shown in the figure. Express your answers in parts (b), (c), and (d) in terms of m_1, m_2, F and D.

(a) (9 Pts.) Draw free body diagrams of two masses and of the pulley.

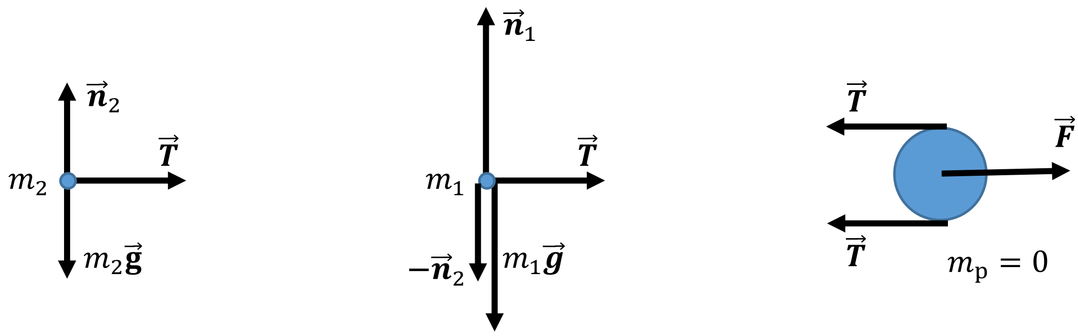
(b) (5 Pts.) What is the tension in the string?

(c) (16 Pts.) Find how long it would take for the mass m_2 to cover the distance D to reach the edge of the platform.

(d) (5 Pts.) Find the acceleration of the pulley with respect to the ground.



Solution: (a)



(b) Since mass of the pulley is zero, we need to have $\Sigma\vec{\mathbf{F}} = 0 \quad \rightarrow \quad 2\vec{\mathbf{T}} + \vec{\mathbf{F}} = 0 \quad \rightarrow \quad \vec{\mathbf{T}} = -\vec{\mathbf{F}}/2$. Hence

$$T = \frac{1}{2}F\,.$$

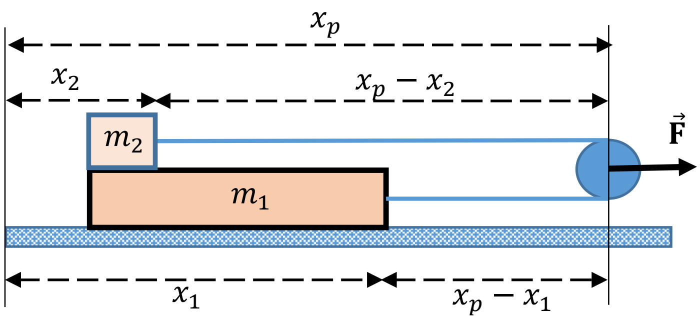
(c) From the free body diagrams, we find $a_2 = T/m_2 = F/(2m_2)$, and $a_1 = T/m_1 = F/(2m_1)$. Relative acceleration of mass 2 with respect to mass 1 is

$$a_{2/1} = a_2 - a_1 = \frac{F}{2}\left(\frac{1}{m_2} - \frac{1}{m_1}\right) = \left(\frac{m_1 - m_2}{2m_1m_2}\right)F\,.$$

Since

$$D = \frac{1}{2}a_{2/1}t^2 \quad \rightarrow \quad t = \sqrt{\frac{2D}{a_{2/1}}} = \sqrt{\frac{2D}{F}\left(\frac{2m_1m_2}{m_1 - m_2}\right)} = \sqrt{\frac{4Dm_1m_2}{F(m_1 - m_2)}}\,.$$

(d) Since the length of the string is constant, we see from the figure that $2x_p - x_1 - x_2 =$ constant. Differentiating twice, we get $2a_p = a_1 + a_2$, where a_p is the acceleration of the pulley, a_1 is the acceleration of m_1, and a_2 is the acceleration of m_2.



This means

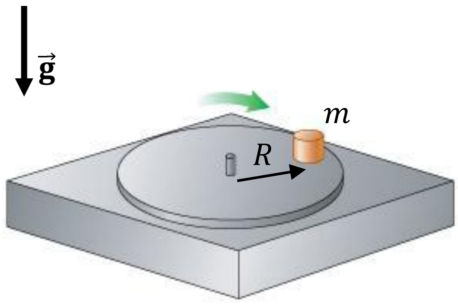
$$a_p = \frac{a_1}{2} + \frac{a_2}{2} = \frac{F}{4m_1} + \frac{F}{4m_2} = \frac{F}{4}\left(\frac{m_1 + m_2}{m_1m_2}\right).$$

3. A small cylinder is at rest with respect to a circular turntable that is rotating at a constant speed with period T. The cylinder is rotating with the turntable, as illustrated in the figure. The cylinder has mass m, the coefficient of static friction between the cylinder and the turntable is μ_s, and the cylinder is located at a distance R from the center of the turntable.

(a) (7 Pts.) What is the acceleration of the cylinder in terms of R and T?

(b) (8 Pts.) What is the magnitude of the friction force acting on the cylinder in terms of m, R and T?

(c) (20 Pts.) If the turntable rotates too fast the mass will start to slide. Find the minimum possible period T_{min} of rotation of the turntable for the mass to be stationary with respect to the turntable.



Solution:

(a) Cylinder rotating with constant speed has radial acceleration

$$a_R = \frac{v^2}{R}, \qquad v = \frac{2\pi R}{T} \quad \rightarrow \quad a_R = 4\pi^2 \frac{R}{T^2}.$$

(b) İt is the friction force which is accelerating the cylinder. Therefore,

$$f = ma_R = 4\pi^2 \frac{mR}{T^2}.$$

(c)

$$n = mg\,, \qquad f \leq \mu_s n \quad \rightarrow \quad f \leq \mu_s mg \quad \rightarrow \quad 4\pi^2 \frac{mR}{T^2} \leq \mu_s mg \quad \rightarrow \quad T^2 \geq \frac{4\pi^2 R}{\mu_s g}.$$

Hence,

$$T_{\mathrm{min}} = 2\pi \sqrt{\frac{R}{\mu_s g}}.$$